

Specimen Paper Answers – Paper 6

Cambridge International AS & A Level Mathematics 9709

For examination from 2020





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Introduction

The main aim of this booklet is to exemplify standards for those teaching Cambridge International AS & A Level Mathematics 9709, and to show examples of model answers to the 2020 Specimen Paper 6. Paper 6 assesses the syllabus content for Probability & Statistics 2. We have provided answers for each question in the specimen paper, along with examiner comments explaining where and why marks were awarded. Candidates need to demonstrate the appropriate techniques, as well as applying their knowledge when solving problems.

Individual examination questions may involve ideas and methods from more than one section of the syllabus content for that component. The main focus of examination questions will be the AS & A Level Mathematics subject content. However, candidates may need to make use of prior knowledge and mathematical techniques from previous study, as listed in the introduction to section 3 of the syllabus.

There are six to eight structured questions in Paper 6; candidates must answer **all** questions. Questions are of varied lengths and often contain several parts, labelled (a), (b), (c), which may have sub-parts (i), (ii), (iii), as needed. Some questions might require candidates to sketch graphs or diagrams, or draw accurate graphs.

Candidates are expected to answer directly on the question paper. All working should be shown neatly and clearly in the spaces provided for each question. New questions often start on a fresh page, so more answer space may be provided than is needed. If additional space is required, candidates should use the lined page at the end of the question paper, where the question number or numbers must be clearly shown.

The mark schemes for the Specimen Papers are available to download from the School Support Hub at www.cambridgeinternational.org/support

2020 Specimen Mark Scheme 6

Past exam resources and other teacher support materials are available on the School Support Hub (<u>www.cambridgeinternational.org/support</u>).

Assessment overview

There are three routes for Cambridge International AS & A Level Mathematics. Candidates may combine components as shown below.

Route 1 AS Level only (Candidates take the AS components in the same series)	Paper 1 Pure Mathematics 1	Paper 2 Pure Mathematics 2	Paper 3 Pure Mathematics 3	Paper 4 Mechanics	Paper 5 Probability & Statistics 1	Paper 6 Probability & Statistics 2
Either	✓				✓	
Or	✓		NI.4	✓		N1.4
Or Note this option in Route 1 cannot count towards A Level	✓	¥	available for AS Level			available for AS Level

Route 2 A Level (staged over two years)	Paper 1 Pure Mathematics 1	Paper 2 Pure Mathematics 2	Paper 3 Pure Mathematics 3	Paper 4 Mechanics	Paper 5 Probability & Statistics 1	Paper 6 Probability & Statistics 2
Either Year 1 AS Level	✓			\checkmark		
Year 2 Complete the A Level			✓		\checkmark	
Or Year 1 AS Level	✓	Not available			\checkmark	
Year 2 Complete the A Level		A Level	✓			√
Or Year 1 AS Level	✓				\checkmark	
Year 2 Complete the A Level			✓	✓		

Route 3 A Level (Candidates take the A Level components in the same series)	Paper 1 Pure Mathematics 1	Paper 2 Pure Mathematics 2	Paper 3 Pure Mathematics 3	Paper 4 Mechanics	Paper 5 Probability & Statistics 1	Paper 6 Probability & Statistics 2
Either	✓	Not available	✓	\checkmark	✓	
Or	1	A Level	~		1	√

Paper 6 – Probability & Statistics 2

- Written examination, 1 hour 15 minutes, 50 marks
- 6 to 8 structured questions based on the Probability & Statistics 2 subject content
- Candidates answer all questions
- Externally assessed by Cambridge International
- 20% of the A Level only

Offered only as part of A Level.

Assessment objectives

The assessment objectives (AOs) are the same for all papers:

AO1 Knowledge and understanding

- Show understanding of relevant mathematical concepts, terminology and notation
- Recall accurately and use appropriate mathematical manipulative techniques

AO2 Application and communication

- Recognise the appropriate mathematical procedure for a given situation
- Apply appropriate combinations of mathematical skills and techniques in solving problems
- Present relevant mathematical work, and communicate corresponding conclusions, in a clear and logical way

Weightings for assessment objectives

The approximate weightings (± 5%) allocated to each of the AOs are summarised below.

Assessment objectives as an approximate percentage of each component

Assessment objective	Weighting in components %					
	Paper 1	Paper 2	Paper 3	Paper 4	Paper 5	Paper 6
AO1 Knowledge and understanding	55	55	45	55	55	55
AO2 Application and communication	45	45	55	45	45	45

Assessment objectives as an approximate percentage of each qualification

Assessment objective	Weighting in AS Level %	Weighting in A Level %
AO1 Knowledge and understanding	55	52
AO2 Application and communication	45	48

- 1 Leaves from a certain type of tree have lengths that are distributed with standard deviation 3.2 cm. A random sample of 250 of these leaves is taken and the mean length of this sample is found to be 12.5 cm.
 - (a) Calculate a 99% confidence interval for the population mean length.

[3]



Examiner comment

The use of a simple sketch often helps clarify the information within the guestion. Although the guestion doesn't state that this is a Normal distribution, the requirement to calculate a confidence interval is an indication of the required approach. The sketch reminds the candidate that for a 99% confidence interval, the z-value of 0.995 must be stated from the critical value section of the tables. **1** The correct *z*-value is awarded the first B1 mark. Stating the formula for the confidence interval is good practice as it minimises the risk of careless substitutions. The values for the mean, standard deviation and sample size are all stated within the guestion – it may be helpful to highlight these on the initial reading of the question. It is reasonable to have a single statement of the two required calculations, as there is a simple sign change, but the calculation should be stated before evaluating; (2) the method mark is awarded when there is an attempt at this. It is good practice to initially state the answer to a greater degree of accuracy than required, so that rounding can be checked by the candidate later.

The confidence interval is awarded the accuracy mark 3. Less mathematical expressions could be used, but credit may not be awarded if no statement is made.

Common errors are using 99% to find the *z*-value of 0.99, or to not use the critical value section of the tables, which may introduce an unacceptable value. It is important to identify the correct values within the information provided, as the use of the variance rather than the standard deviation is another common error.

Write down the probability that the whole of a 99% confidence interval will lie below the population mean.	[1]	
		Examiner comment
p = 0.005		This is a simple interpretation of the information, and the earlier diagram helps to identify the required probability area – the tail to the left of the mean. Stating the value is required to gain the mark. The most frequent error in this kind of question is to assume that, as the question references the confidence interval to be less than the mean, then the probability will be 0.5.



- **3** The number of calls received at a small call centre has a Poisson distribution with mean 2.4 calls per 5-minute period.
 - (a) Find the probability of exactly 4 calls in an 8-minute period.

[2]

2.4 calls in 5 minutes, so $\lambda = \frac{2.4}{5} \times 8 = 3.84$ calls expected in 8 minutes.

Po(3.84) is required.

 $P(4 \text{ calls}) = e^{-3.84} \times \frac{3.84^4}{4!}$ 1 = 0.1947 = 0.195

Examiner comment

The solution starts by calculating the expected number of calls in the 8-minute period using the properties of the Poisson distribution. It is helpful to state the distribution that is being used as this clearly identifies the values required in the formula. The method mark is awarded when the correct values are substituted into the formula 1. Efficient use of the calculator will produce the answer, which has been rounded to the expected degree of accuracy.

Common errors include failing to evaluate the calculations accurately, including the simple proportion calculation.



Examiner comment

The solution starts by calculating the expected number of calls in the 3-minute period using the properties of the Poisson distribution. Stating the distribution is helpful to clearly identify the value to be used within the calculations.

The requirement of 'at least 3 calls' is interpreted correctly as 'not having 0, 1 or 2 calls'. The numerical statement **1** is awarded a method mark as an indication of the correct process, and a further method mark is awarded for using the correct values. The final answer will gain the accuracy mark, stated to the anticipated degree of accuracy **2**.

The most common error in this type of question is the misinterpretation of 'at least', with 3 calls often being omitted from the required probability.

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The number of calls received at a large call centre has a Poisson distribution with mean 41 calls per 5-minute period.

(c) Use an approximating distribution to find the probability that the number of calls received in a 5-minute period is between 41 and 59 inclusive.



[5]

Examiner comment

The candidate has stated the Normal distribution as appropriate to use, identifying the mean and variance . This is sufficient for the first B1 mark. There is then consideration of the nature of the data and whether a continuity correction is required. The statement of the probability calculation required clarifies the process needed.

For 59 to be included, the continuity correction must be added, and for 41 to be included, it must be subtracted. Each statement of the standardisation formula with the values substituted is awarded a method mark 2. Simple sketches help to identify the correct area required for the probability, which when attempted gains the final method mark 3. The statement of the answer to an appropriate degree of accuracy finishes the question for the final mark 4.

A frequent error is failing to apply the continuity correction correctly, or treating the data as continuous and not requiring a continuity correction. The use of the variance in the formula continues to be an issue for some candidates. The use of simple diagrams is to be recommended as it helps to clarify the required area, as there is often confusion as to which values to use.



(a) Find the probability that the total of the lifetimes of five randomly chosen Longlive bulbs is less than 5200 hours.



[4]

Examiner comment

The question states that both light bulbs have independent Normal distributions for their lifetimes, so the candidate uses the Normal distribution properties to calculate the new mean and variance for the five light bulbs; 1 these values are awarded a B1 mark. It is not necessary to evaluate at this stage.

The use of the standardisation formula is indicated by the question, and substituting in the correct values will gain the first method mark 2. At this stage the candidate evaluates the formula, as this minimises the possibility of premature approximation errors. The value has been written to 4SF as this is a working value. Consideration of the required probability area is needed. Using the correct area gains the final method mark 3 which leads to the accuracy mark at the final answer 4.

Understanding the context is essential in this type of question, as the most common errors occur where candidates have not interpreted the data correctly. A common error in this kind of question is using a squared coefficient for the variance.



Examiner comment

The candidate starts by changing the context into a 'variable' so that the properties of the Normal distribution can be used. At this stage, the first method mark is awarded 1. To use the standardisation formula, the expected value 2 and variance 3 for the new variable must be calculated. Stating the calculations required will enable these to be checked later. These are each awarded a B1 mark.

The standardisation formula is applied using 0 as the key value. When the calculated mean and standard deviation are substituted, a further method mark is awarded 4. Consideration of the required area is needed. The simple sketch clarifies this, with the final method mark being awarded when the correct calculation is stated 5. Correct use of the tables leads to the final mark being awarded 6.

Common errors in this kind of question occur with the application of the Normal distribution properties, particularly within the variance. Failure to square the coefficient is another common error. Modelling the mathematical context is challenging, but is a key skill required for this type of question.



(b) State the value of $P(1.5 \le X \le 4)$. [1] As E(X) = 1.5, $P(1.5 \le X \le 4) = 0.5$ (c) Given that $P(1 \le X \le 2) = \frac{13}{27}$, find P(X > 2). [2] 1.5 2 З 1 $P(X > 2) = \frac{1 - P(1 \le X \le 2)}{2}$ $=\frac{1-\frac{13}{27}}{2} = \frac{7}{27}$

Examiner comment

The lower limit of the required probability is the expected value, and the range extends into the '0 value' section of the probability density function. The answer is stated for the B1 mark.

Examiner comment

A simple sketch helps to clarify the question. This identifies that the upper unshaded section is required and the logic to calculate is stated. 1 When the values are substituted, the method mark is awarded. The final answer is stated as a fraction, which is an exact value and requires no further conversion to be awarded the accuracy mark 2.

Candidates can take an alternative approach to this question, and use $\int_{2}^{3} (3x - x^2) dx$. This would gain full credit, but is a more complex approach. Many candidates find modelling the context challenging here, and the use of the simple diagram presents the information visually.



Examiner comment

The candidate identifies why a one-tailed test is appropriate 1. As the probability of not arriving was 0.2 and the intention is to make a difference to this, the candidate states a null and alternative hypothesis to reflect the impact expected 2. Independent marks were awarded for each of 1 and 2. Using the context when explaining why a process is appropriate is often essential for credit to be awarded. A general statement may not be sufficient.

Mixing up the null and alternative hypothesises is a common error seen in this type of question.

(b) Use a binomial distribution to find the critical region, and find the probability of a Type I error.

 $P(X \le 1) = 0.8^{30} + {}^{30}C_1 \ 0.8^{29} \times 0.2$ $P(X \le 2) = P(X \le 1) + {}^{30}C_2 \ 0.8^{28} \times 0.2^2 = 0.04418$ $P(X \le 3) = P(X \le 2) + {}^{30}C_3 \ 0.8^{27} \times 0.2^3 = 0.1227 \text{ (greater than 5\%)}$ So at the 5% significance level, the critical region is $X \le 2$. $P(Type \ 1 \ error) = 0.0442$

Examiner comment

[5]

The candidate is taking a systematic approach to finding the critical region. This also enables the efficient use of a calculator, as the preceding answer can be used in each new term. By presenting the unsimplified binomial expression for each probability, the candidate is communicating clearly their logic, and attempting P(X = 0) and P(X = 1) is sufficient to gain a method mark **1**. Although a 4SF answer is stated for using a calculator efficiently, the more accurate answer should be held on the calculator and used for calculating $P(X \le 2)$ 2. A further method mark is awarded here. As the answer is still less than 0.05, the candidate calculates $P(X \le 3)$ 3. A B1 mark is awarded for this value. The candidate states that this is greater than the 5% significance level required. The candidate then states the critical region 4 which, as it is supported by working, is awarded an accuracy mark. There is also a statement of the required probability **5**, which is also awarded an accuracy mark as supported by the previous workings.

Many candidates do not work systematically when identifying critical regions. The omission of lower values means that the evidence is incomplete and full credit may not be awarded. Errors with accuracy of values occur because candidates state, and then use, 3 significant figure answers at each stage. The failure to state the answer to the actual question is a common omission. (c) In fact 3 patients out of the 30 do not arrive. State the conclusion of the test, explaining your answer.
There is no evidence that the publicity has worked in reducing missed appointments 1 as 3 is outside of the Critical Region. 2

Examiner comment

[2]

The candidate indicates their conclusion from the test 1 but no mark is awarded until it is justified by the reason provided 2. The reason would gain the method mark, and the previous statement the accuracy mark.

Failure to use the context in reasons is a common error in this kind of question. Hypotheses tests seek to interpret real-life situations and conclusions should be within the context given.

7 The mean weight of bags of carrots is μ kilograms. An inspector wishes to test whether $\mu = 2.0$. He weighs a random sample of 200 bags and his results are summarised as follows.

 $\Sigma x = 430$ $\Sigma x^2 = 1290$

Carry out the test at the 10% significance level.



Examiner comment

The candidate identifies that this is a hypothesis test, so states an appropriate null and alternative hypotheses initially, this gains a B1 mark **1** The sample mean is calculated which is awarded a B1 mark (2) The candidate states the formula to calculate the unbiased estimate of the population variance. On substituting the appropriate values 3 a method mark is awarded. Stating the formula helps eliminate inaccurate calculations. The answer is stated to 8SF although credit would be given here for answers to at least 4SF. Because of the size of the population, a normal approximation is appropriate to use to calculate the required probability. The candidate uses the standardisation formula with their population mean and variance 5. This is awarded a further method mark. Although required, no mark is awarded for the numerical evaluation in this guestion. The candidate then interprets the 10% significance level as requiring the critical value 0.95 and uses tables to state the required *z*-value 6 This is awarded a final method mark. The candidate then makes a final conclusion about the test the inspector has performed **7** for a final accuracy mark.

[7]

Common errors include mixing up the null and alternative hypothesis when interpreting the context of the question. Answers may also be affected by early approximation within the calculation. Candidates are advised to use their calculators efficiently and to use accurate values throughout. Care also needs to be taken to identify whether the biased or unbiased population variance formula should be used.

Cambridge Assessment International Education The Triangle Building, Shaftesbury Road, Cambridge, CB2 8EA, United Kingdom t: +44 1223 553554 e: info@cambridgeinternational.org www.cambridgeinternational.org