* 0000800000001 *

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

* 5 7 8 4 5 1 3 0 3 2 *

FURTHER MATHEMATICS **9231/11**

Paper 1 Further Pure Mathematics 1 **May/June 2025**

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

DC (CE) 355890

1 **(a)** Use standard results from the list of formulae (MF19) to show that

$$\sum_{r=1}^{n}(2-3r)(5-3r) = an^3 + bn^2 + cn,$$

where a, b and c are integers to be determined. [3]

(b) Use the method of differences to find $\sum_{r=1}^{n} \frac{1}{(2-3r)(5-3r)}$ in terms of n. [4]

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(c) Deduce the value of $\sum_{r=1}^{\infty} \frac{1}{(2-3r)(5-3r)}$. [1]

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2 The cubic equation $x^3 + 2x + 1 = 0$ has roots α, β, γ.

(a) Find a cubic equation whose roots are $\alpha^3 - 1,\ \beta^3 - 1,\ \gamma^3 - 1$. [3]

(b) Find the value of $(\alpha^3 - 1)^2 + (\beta^3 - 1)^2 + (\gamma^3 - 1)^2$. [2]

(c) Find the value of $(\alpha^3 - 1)^3 + (\beta^3 - 1)^3 + (\gamma^3 - 1)^3$. [2]

3 The sequence $u_1, u_2, u_3, \ldots$ is such that $u_1 = 5$ and $u_{n+1} = 6u_n + 5$ for $n \geqslant 1$.

(a) Prove by induction that $u_n = 6^n - 1$ for all positive integers n. [5]

(b) Deduce that u_{2n} is divisible by u_n for $n \geqslant 1$. [2]

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4 The matrix $\mathbf{M}$ is given by $\mathbf{M} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$, where $0 < \theta < 2\pi$.

(a) The matrix $\mathbf{M}$ represents a sequence of two geometrical transformations in the x–y plane.

State the type of each transformation, and make clear the order in which they are applied. [2]

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(b) Find the value of θ for which the transformation represented by $\mathbf{M}$ has a line of invariant points. [7]

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5 The curve C has polar equation $r = \theta e^{\frac{1}{8}\theta}$, for $0 \leqslant \theta \leqslant 2\pi$.

(a) Sketch C. [2]

(b) Find the area of the region bounded by C and the initial line, giving your answer in the form $\left(p\pi^2 + q\pi + r\right)e^{\frac{1}{2}\pi} + s$, where p, q, r and s are integers to be determined. [6]

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(c) Show that, at the point of C furthest from the initial line,

$$\theta\cos\theta+\left(\tfrac{1}{8}\theta+1\right)\sin\theta=0$$

and verify that this equation has a root between 5 and 5.05. [5]

6 The points A, B, C have position vectors

$$\mathbf{i} - 2\mathbf{k}, \quad \mathbf{i} + 2\mathbf{j} + 2\mathbf{k}, \quad 2\mathbf{i} - \mathbf{j} - \mathbf{k},$$

respectively.

(a) Find the equation of the plane ABC, giving your answer in the form $ax + by + cz = d$. [5]

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A point D has position vector $\mathbf{i} + t\mathbf{k}$, where $t \neq -2$.

(b) Find the acute angle between the planes ABC and ABD. [4]

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(c) Find the values of t such that the shortest distance between the lines AB and CD is $\sqrt{2}$. [7]

7 The curve C has equation $y = \dfrac{2x^2 - 5x}{2x^2 - 7x - 4}$.

(a) Find the equations of the asymptotes of C. [2]

(b) Find the coordinates of any stationary points on C. [4]

(c) Sketch C, stating the coordinates of the intersections with the axes. [3]

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(d) Sketch the curve with equation $y = \left| \dfrac{2x^2 - 5x}{2x^2 - 7x - 4} \right|$. [1]

(e) Find in exact form the set of values of x for which $\left|\frac{2x^2-5x}{2x^2-7x-4}\right| < \frac{1}{9}$. [5]

Additional page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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