* 0000800000001 *

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

* 1 2 2 7 5 3 4 7 0 3 *

FURTHER MATHEMATICS **9231/13**

Paper 1 Further Pure Mathematics 1 **May/June 2025**

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

DC (LK) 340754/2

1 The matrix $\mathbf{M}$ represents the sequence of two transformations in the x-y plane given by a stretch parallel to the x-axis, scale factor 14, followed by a rotation anticlockwise about the origin through angle $\frac{1}{3}\pi$.

(a) Show that $2\mathbf{M} = \begin{pmatrix} 14 & -\sqrt{3} \\ 14\sqrt{3} & 1 \end{pmatrix}$. [4]

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(b) Find the equations of the invariant lines, through the origin, of the transformation in the x-y plane represented by $\mathbf{M}$. [5]

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The unit square S in the x-y plane is transformed by $\mathbf{M}$ onto the rectangle P.

(c) Find the matrix which transforms P onto S. [2]

2 Prove by mathematical induction that $2025^n + 47^n - 2$ is divisible by 46 for all positive integers n. [6]

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3 The quartic equation $x^4 + 7x^2 + 3x + 22 = 0$ has roots $\alpha, \beta, \gamma, \delta$.

(a) Find the value of $\alpha^2 + \beta^2 + \gamma^2 + \delta^2$. [2]

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(b) Find the value of $\alpha^4 + \beta^4 + \gamma^4 + \delta^4$. [2]

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(c) Use standard results from the list of formulae (MF19) to find the value of

$$\sum_{r=1}^{10}\left(\left(\alpha^2+r\right)^2+\left(\beta^2+r\right)^2+\left(\gamma^2+r\right)^2+\left(\delta^2+r\right)^2\right).$$ [5]

4 Let $w_r = r(r+1)(r+2)\ldots(r+9)$.

(a) Show that

$$w_{r+1} - w_r = 10(r+1)(r+2)\ldots(r+9).$$ [2]

(b) Given that $u_r = (r+1)(r+2)\ldots(r+9)$, find $\sum_{r=1}^{n} u_r$ in terms of n. [3]

(c) Given that $v_r = x^{w_{r+1}} - x^{w_r}$, find the set of values of x for which the infinite series

$$v_1 + v_2 + v_3 + \dots$$

is convergent and give the sum to infinity when this exists. [3]

5 The plane Π has equation $\mathbf{r} = 2\mathbf{i} + 3\mathbf{j} - 2\mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} - \mathbf{k}) + \mu(3\mathbf{i} + 2\mathbf{j} - 2\mathbf{k})$.

(a) Find a Cartesian equation of Π, giving your answer in the form $ax + by + cz = d$. [4]

The point P has position vector $4\mathbf{i} + 2\mathbf{j} + 9\mathbf{k}$.

(b) Find the position vector of the foot of the perpendicular from P to Π. [4]

(c) The line l is parallel to the vector $3\mathbf{i} + 5\mathbf{j} - \mathbf{k}$.

Find the acute angle between l and Π. [3]

6 The curve C has equation $y = \dfrac{x^2 + a}{x + a}$, where a is a positive constant.

(a) Find the equations of the asymptotes of C. [3]

(b) Find, in terms of a, the x-coordinates of the stationary points on C. [3]

(c) Sketch C, stating the coordinates of any intersections with the axes. [3]

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(d) Sketch the curve with equation $y = \left|\frac{x^2+a}{x+a}\right|$. [1]

(e) Find the set of values of a for which $\left|\dfrac{x^2+a}{x+a}\right| = a$ has two real solutions. [4]

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7 The curve C has polar equation $r^2 = e^{\sin\theta}\cos\theta$, for $-\frac{1}{2}\pi \leqslant \theta \leqslant \frac{1}{2}\pi$.

(a) Find the polar coordinates of the point on C that is furthest from the pole, giving your answers correct to 3 decimal places. [5]

(b) Find the polar coordinates of the point on C that is furthest from the half-line $\theta = \frac{1}{2}\pi$, giving your answers correct to 3 decimal places. [5]

(c) Sketch C. [3]

(d) Find the area of the region bounded by C, giving your answer in exact form. [3]

Additional page

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