Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

FURTHER MATHEMATICS **9231/14**

Paper 1 Further Pure Mathematics 1 **May/June 2025**

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages.

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1 **(a)** Use the List of formulae (MF19) to find $\sum_{r=1}^{n}(2r+1)$ in terms of n, simplifying your answer. [2]

(b) Show that $\dfrac{2r+1}{(r^2+1)(r^2+2r+2)} = \dfrac{1}{r^2+1} - \dfrac{1}{r^2+2r+2}$. [1]

(c) Use the method of differences to find $\sum_{r=1}^{n} \frac{2r+1}{(r^2+1)(r^2+2r+2)}$. [2]

(d) Deduce the value of $\sum_{r=1}^{\infty} \frac{2r+1}{(r^2+1)(r^2+2r+2)}$. [1]

2 Prove by mathematical induction that, for every integer $n \geqslant 2$,

$$\frac{\mathrm{d}^n}{\mathrm{d}x^n}(x \ln x) = (-1)^n (n-2)\,! x^{1-n}.$$ [6]

3 The points A, B and C have position vectors

$$2\mathbf{j}+3\mathbf{k}, \quad -5\mathbf{i}+3\mathbf{j}+\mathbf{k} \quad \text{and} \quad \mathbf{i}+2\mathbf{j}+5\mathbf{k}$$

respectively, relative to the origin O.

(a) Find the equation of the plane ABC, giving your answer in the form $ax+by+cz=d$. [5]

(b) Find the perpendicular distance from O to the plane ABC. [2]

(c) Find the acute angle between the line OA and the plane ABC. [3]

4 The cubic equation $x^3 + bx^2 + cx - 1 = 0$, where b and c are constants, has roots α, β, γ.

It is given that the matrix $\begin{pmatrix} 1 & \alpha & \beta \\ \alpha & 1 & \gamma \\ \beta & \gamma & 1 \end{pmatrix}$ is singular.

(a) Show that $\alpha^2 + \beta^2 + \gamma^2 = 3$. [4]

(b) It is given that $\alpha^3 + \beta^3 + \gamma^3 = 3$ and that the constants b and c are positive.

Find the values of b and c. [6]

5 The matrix **M** represents a sequence of two transformations in the x-y plane given by a one-way stretch in the x-direction, scale factor 3, followed by a reflection in the line $y = x$.

(a) Find **M**. [3]

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(b) Give full details of the geometrical transformation in the x-y plane represented by $\mathbf{M}^{-1}$. [3]

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The matrix **N** is such that $\mathbf{MN} = \begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix}$.

(c) Find **N**. [3]

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(d) Find the equations of the invariant lines, through the origin, of the transformation in the x-y plane represented by **MN**. [5]

6 The curve C has polar equation $r = a\tan\left(\frac{1}{8}\theta\right)$, where a is a positive constant and $0 \leqslant \theta \leqslant 2\pi$.

(a) Sketch C and state, in terms of a, the greatest distance of a point on C from the pole. [3]

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(b) Find, in terms of a, the area of the region bounded by C and the initial line. [4]

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(c) Show that, at the point on C furthest from the initial line,

$$4\sin\left(\tfrac{1}{4}\theta\right)\cos\theta + \sin\theta = 0$$

and verify that this equation has a root between 4.95 and 5. [6]

7 The curve C has equation $y = \dfrac{x^2+x-4}{x^2+x+2}$.

(a) State the equation of the asymptote of C. [1]

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(b) Show that, for all real values of x, $-\frac{17}{7} \leqslant y < 1$. [4]

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(c) Find the coordinates of any stationary points of C. [3]

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(d) Sketch C, stating the coordinates of the intersections with the axes. [3]

(e) Sketch the graph with equation $y = \dfrac{|x|^2 + |x| - 4}{|x|^2 + |x| + 2}$ and find the set of values of x for which

$\dfrac{|x|^2 + |x| - 4}{|x|^2 + |x| + 2} < -\frac{1}{2}$. [5]

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Additional page

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