* 0000800000001 *

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME						
CENTRE NUMBER						
CANDIDATE NUMBER						

* 0 3 0 1 2 9 1 1 2 1 *

FURTHER MATHEMATICS **9231/21**

Paper 2 Further Pure Mathematics 2 **May/June 2025**

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

1 Find the roots of the equation $z^3 = 27 - 27\mathrm{i}$, giving your answers in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$ and $-\pi \leqslant \theta < \pi$. [5]

2 Let $I_n = \int_0^1 (1-x)^n \sinh x \, dx$, where n is a non-negative integer.

(a) Show that, for $n \geqslant 2$, $I_n = -1 + n(n-1)I_{n-2}$. [4]

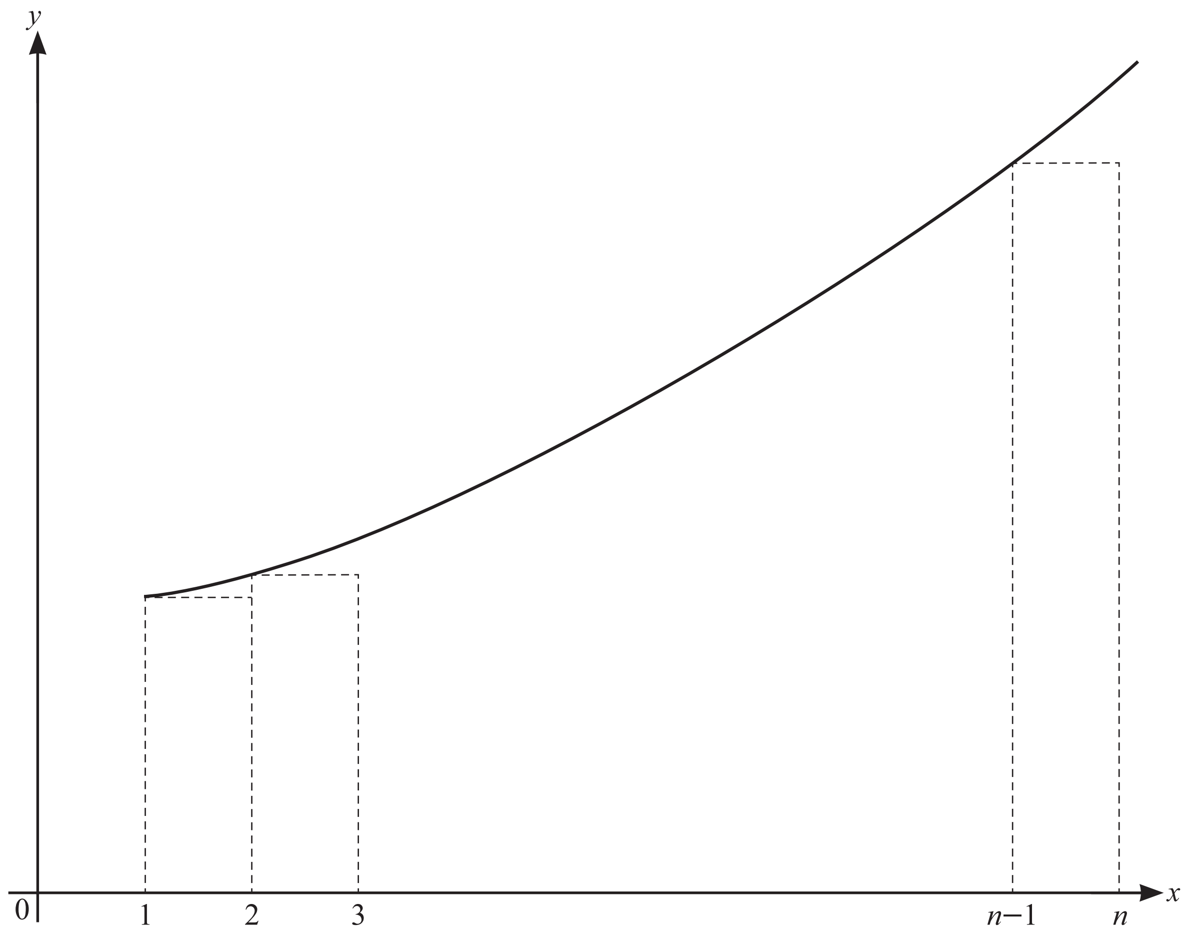
(b) Find the exact value of I_2. [3]

3 By considering the binomial expansion of $\left(z-\dfrac{1}{z}\right)^5$, where $z=\cos\theta+\mathrm{i}\sin\theta$, use de Moivre's theorem to show that

$$\operatorname{cosec}^5\theta=\frac{a}{\sin 5\theta+b\sin 3\theta+c\sin\theta},$$

where a, b and c are integers to be determined. [6]

4



The diagram shows the curve with equation $y = \frac{1}{\sqrt{x}}e^{\sqrt{x}}$ for $x \geqslant 1$, together with a set of $n-1$ rectangles of unit width.

(a) By considering the sum of the areas of these rectangles, show that

$$\sum_{r=1}^{n} \frac{1}{\sqrt{r}} e^{\sqrt{r}} < \left(2 + \frac{1}{\sqrt{n}}\right) e^{\sqrt{n}} - 2e.$$ [5]

....

....

....

....

....

....

....

....

....

(b) Use a similar method to find, in terms of n, a lower bound for $\sum_{r=1}^{n} \frac{1}{\sqrt{r}} e^{\sqrt{r}}$. [4]

5 Find the particular solution of the differential equation

$$6\frac{d^2x}{dt^2} + 3\frac{dx}{dt} + 6x = e^{-t},$$

given that, when $t = 0$, $x = \frac{dx}{dt} = 0$. [10]

6 **(a)** Starting from the definitions of tanh and sech in terms of exponentials, prove that

$$1-\tanh^2 u = \operatorname{sech}^2 u.$$ [3]

....

....

....

....

....

....

....

....

....

....

....

....

(b) Show that $\frac{\mathrm{d}}{\mathrm{d}t}(\operatorname{sech}^{-1} t) = -\frac{1}{t\sqrt{1-t^2}}$. [4]

....

....

....

....

....

....

....

....

....

....

....

....

It is given that

$$x = \tanh^{-1} t \quad \text{and} \quad y = t\,\text{sech}^{-1} t, \quad \text{for } 0 < t < 1.$$

(c) Show that $\dfrac{dy}{dx} = -\sqrt{1-t^2} + (1-t^2)\,\text{sech}^{-1} t$. [4]

(d) Find $\dfrac{d^2y}{dx^2}$ in terms of t. [4]

7 Find the solution of the differential equation

$$\frac{dy}{dx} - \frac{x+5}{x^2+10x+61}y = 1,$$

given that $y = 0$ when $x = 3$. Give your answer in an exact form. [10]

8 **(a)** It is given that λ is an eigenvalue of the non-singular square matrix **A**, with corresponding eigenvector **e**.

Show that **e** is an eigenvector of $\mathbf{A}^3$ with corresponding eigenvalue λ^3. [2]

....

....

....

....

....

....

....

....

The matrix **A** is given by

$$\mathbf{A} = \begin{pmatrix} -1 & 3 & 4 \\ 0 & 1 & 0 \\ 0 & -2 & 5 \end{pmatrix}.$$

(b) Show that the eigenvalues of **A** are -1, 1 and 5. [2]

....

....

....

....

....

....

....

....

....

....

....

....

....

(c) Find a matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{A}-2\mathbf{I}=\mathbf{PDP}^{-1}$. [6]

(d) Use the characteristic equation of $\mathbf{A}$ to show that $(\mathbf{A}-2\mathbf{I})^3 = a\mathbf{A}^2+b\mathbf{A}+c\mathbf{I}$ where a, b and c are constants to be determined. [3]

Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

BLANK PAGE

BLANK PAGE

BLANK PAGE

BLANK PAGE