Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

FURTHER MATHEMATICS **9231/32**

Paper 3 Further Mechanics **May/June 2025**

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use $10\,\mathrm{m\,s^{-2}}$.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

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1 A particle P of mass 8 kg is moving in a straight horizontal line. At time t s, P has displacement x m from a fixed point O on the line and velocity v m s^{-1}. The only horizontal force acting on P has magnitude $(x^3 + 4x)$ N and acts in the direction OP. When $t = 0$, $x = 0$ and $v = 1$.

(a) Find an expression for v in terms of x, giving your answer in the form $v = ax^2 + b$, where a and b are constants to be determined. [3]

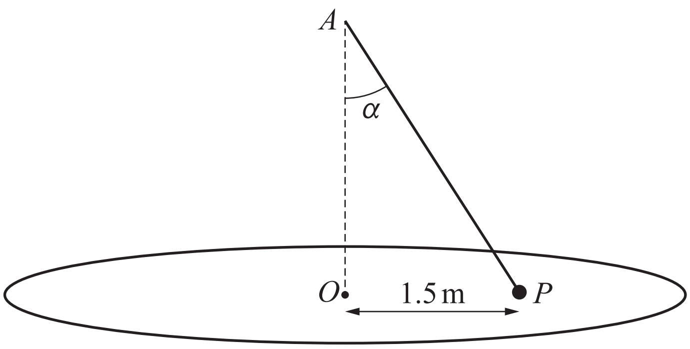
(b) Find an expression for x in terms of t. [2]

2 A particle P is projected with speed $24\,\text{m}\,\text{s}^{-1}$ at an angle $\theta°$ above the horizontal from a point O on a horizontal plane, and moves freely under gravity. At a horizontal distance 35 m from O, there is a vertical wall of height 10 m which is perpendicular to the vertical plane of motion of P.

(a) Determine the two values of θ for which P just clears the wall. [4]

(b) Given that P clears the wall, find the minimum distance from point O where P can land. [2]

3



A rough horizontal disc, centre O, rotates with constant angular speed $\omega\ \text{rad s}^{-1}$. A particle P of mass 1.6 kg lies on the disc at a distance 1.5 m from O, and is attached to a point A vertically above O by a light elastic string. The string has natural length 2 m, modulus of elasticity 32 N and makes an angle α with the vertical OA (see diagram). Particle P moves in a horizontal circle also at a constant angular speed $\omega\ \text{rad s}^{-1}$. Particle P is on the point of slipping in the direction OP. The coefficient of friction between the particle and the disc is 0.5.

(a) Given that the tension in the string is 8 N, show that $\sin\alpha = 0.6$. [2]

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(b) Find the number of revolutions per minute made by the disc and the particle P. [5]

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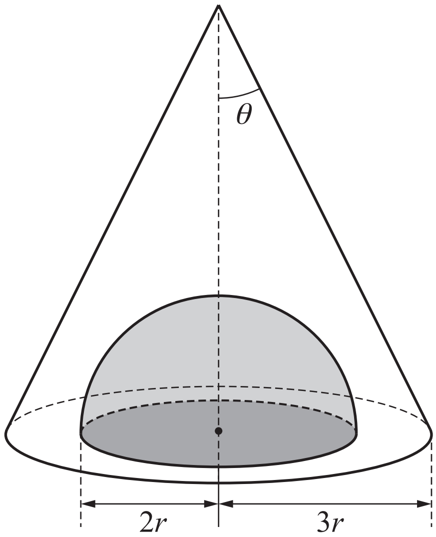

An object is formed by removing a solid hemisphere, radius $2r$, from a uniform solid cone, radius $3r$ and semi-vertical angle θ, where $\tan\theta = \frac{1}{2}$. The axes of symmetry of the cone and the hemisphere coincide. The base of the cone and the base of the hemisphere are in the same plane as each other (see diagram).

(a) Find, in terms of r, the distance of the centre of mass of the object from its base. [4]

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The object is placed such that its circular base makes contact with a rough plane which is inclined to the horizontal at an angle α. The object is on the point of toppling. The plane is sufficiently rough to prevent sliding.

(b) Find the value of α. [3]

5 One end of a light elastic string of natural length 0.5 m and modulus of elasticity 14 N is attached to a fixed point A on a smooth plane. The plane makes an angle α to the horizontal, where $\tan\alpha = \frac{7}{24}$. A particle P of mass 2 kg is attached to the other end of the string. The string lies along a line of greatest slope of the plane. The particle P is initially held on the plane above the level of A, where $AP = 0.8$ m. The particle P is then released from rest.

Find the maximum velocity of P during the subsequent motion. [6]

6 Two uniform smooth spheres A and B of equal radii have masses $2m$ and m respectively. Sphere A is moving in a straight horizontal line with speed u, and sphere B is stationary. Sphere A collides directly with B, and they both then move in the same direction with speeds v_A and v_B respectively. After the collision, the kinetic energy of B is $\frac{9}{2}$ times the kinetic energy of A.

(a) Show that $v_B = \frac{6}{5}u$. [3]

Sphere B then collides with a fixed vertical barrier. Immediately before the collision, the direction of motion of B makes an angle α with the barrier. Immediately after the collision, the direction of motion of B makes an angle β with the barrier. The coefficient of restitution between B and the barrier is $\frac{4}{5}$. As a result of the collision, the velocity of B is reduced to $\frac{12}{25}\sqrt{5}u$.

(b) Find the value of $\sin(\alpha + \beta)$. [6]

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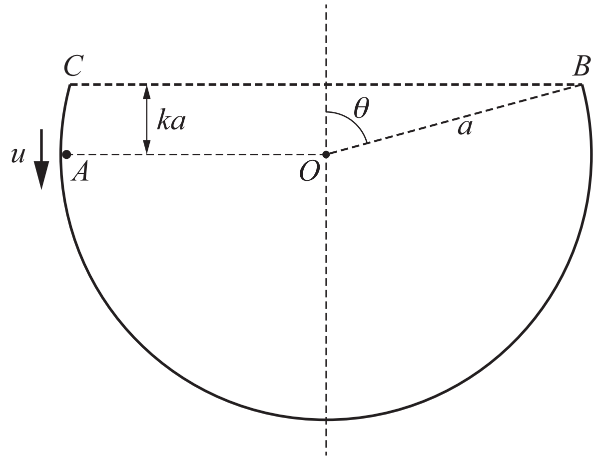

A fixed hollow sphere has radius a and centre O. The points A, B and C lie on the inner surface of the sphere with OA and BC horizontal. A portion of the sphere has been removed by a horizontal cut through points B and C at a vertical distance ka above the centre of the sphere, where k is a positive constant and $k < 1$. The points O, A, B and C all lie in the same vertical plane. OB makes an angle θ with the upward vertical through O (see diagram).

A particle P of mass m is free to move on the smooth inner surface of the sphere. The particle P is projected vertically downwards from A with speed u and begins to move in a vertical circle.

(a) In the case where $u = \sqrt{\frac{6}{5}ga}$, the reaction on P at B is half the reaction on P at A.

Find the value of k. [5]

(b) Find an expression for u, in terms of a and g, in the case that the particle just reaches B. [1]

(c) Find an expression for u, in terms of a and g, in the case that the particle passes through B and in its subsequent motion reaches C. [4]

Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.