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Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

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FURTHER MATHEMATICS **9231/41**

Paper 4 Further Probability & Statistics **May/June 2025**

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

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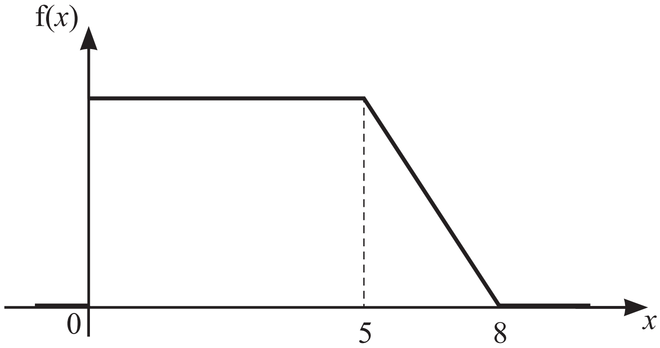
1 The manager of a hardware store is interested in whether there is a difference in the amount spent per customer on weekdays ($\$x$) compared to weekends ($\$y$). Random samples of 120 customers on weekdays and 80 customers on weekends are taken and the amount spent by each customer is recorded. The results are summarised as follows.

$$\Sigma x = 10\,470 \qquad \Sigma(x-\bar{x})^2 = 12\,283 \qquad \Sigma y = 6560 \qquad \Sigma(y-\bar{y})^2 = 13\,520$$

Test at the 1% significance level whether there is a difference in the mean amount spent per customer on weekdays compared to weekends. You should not assume that the population variances of the amounts spent on weekdays and weekends are equal. [7]

2



As shown in the diagram, the continuous random variable X has probability density function f given by

$$\mathrm{f}(x) = \begin{cases} a & 0 \leqslant x \leqslant 5, \\ b - cx & 5 \leqslant x \leqslant 8, \\ 0 & \text{otherwise,} \end{cases}$$

where a, b and c are constants.

(a) Show that $a = \frac{2}{13}$ and find the values of b and c. [3]

(b) Find the mean of X. [3]

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(c) Find the median of X. [2]

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The random variable Y is defined by $Y = X^2$.

(d) Find the cumulative distribution function for Y. [4]

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3 Eggs in a supermarket are sold in boxes of six. A supermarket manager wishes to model the number of broken eggs in the boxes sold in the store. A random sample of 2000 boxes is taken and the number of broken eggs recorded. The observed frequencies are shown in the table below.

Number of broken eggs	0	1	2	3	4	5	6
Observed frequency	1844	143	11	0	1	0	1

(a) Use the data to estimate the probability that an egg is broken. Give your answer correct to 4 significant figures. [1]

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It is decided to carry out a goodness of fit test at the 0.5% significance level to determine whether a binomial distribution fits the data.

The observed frequencies and the expected frequencies are given in the following table.

Number of broken eggs	0	1	2	3	4	5	6
Observed frequency	1844	143	11	0	1	0	1
Expected frequency	1831.3	a	6.016	0.119	0.001	0.000	0.000

(b) Show that $a = 162.6$ correct to 1 decimal place. [1]

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(c) Carry out a goodness of fit test at the 0.5% level of significance to test whether a binomial distribution is a satisfactory model for the data. [5]

(d) Give a reason why a binomial distribution may not be a suitable model in this situation. [1]

4 A researcher is interested in whether there is a difference between two schools in students' aptitude for English. She randomly chooses 10 students from school X and 8 students of a similar age from school Y to take a written English test. The scores for the students from school X (x) and school Y (y) are summarised as follows.

$$\sum x = 612 \qquad \sum x^2 = 40\,104 \qquad \sum y = 444 \qquad \sum y^2 = 27\,460$$

You should assume that the two distributions are normal and have the same population variance.

(a) Find a 95% confidence interval for the difference in the mean scores for students from school X and students from school Y in the written English test. [6]

(b) Use the confidence interval you found in part **(a)** to explain why there is insufficient evidence at a 5% significance level to suggest that the English scores of students from school X and students from school Y are different. [1]

5 Two jigsaw puzzles have the same number of pieces with identical shapes but have different pictures printed on them. One puzzle has a seaside picture and the other has a cartoon picture. A researcher believes that children will complete the cartoon puzzle more quickly. To test this belief, 10 children are randomly selected. The time taken in seconds for each child to complete each puzzle is recorded below.

Child	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*
Seaside	182	130	193	181	192	204	184	192	180	189
Cartoon	161	111	195	159	202	200	168	165	145	160

(a) Carry out a Wilcoxon matched-pairs signed-rank test at the 5% significance level to test the researcher's belief. [6]

(b) Show that using a paired-sample sign test at the 5% significance level would result in the opposite conclusion to that found in part **(a)**. [3]

It was later discovered that the experiment had been conducted such that each child completed the seaside puzzle first followed by the cartoon puzzle.

(c) Comment on the validity of using this experiment to test the researcher's belief. [1]

6 Y is a discrete random variable which takes the values 0, 2, 4, … . The probability generating function of Y is given by

$$G_Y(t) = \frac{k}{1 - at^2}.$$

(a) Find k in terms of a. [1]

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(b) Show that $P(Y > 2) = a^2$. [3]

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It is now given that $a = 0.2$.

(c) Find the value of $\mathrm{E}(Y)$. [2]

Additional page

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