Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

FURTHER MATHEMATICS **9231/43**

Paper 4 Further Probability & Statistics **May/June 2025**

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **12** pages.

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[Turn over

1 A person's eye colour may be categorised as "brown", "blue" or "other". A scientist claims that these eye colours are uniformly distributed and hence are equally likely to occur in the population. A survey of 120 people from this population found that 38 people had brown eyes, 52 people had blue eyes and 30 people had eyes which were neither brown nor blue.

Use the data to carry out a goodness of fit test at the 5% significance level to test the scientist's claim. [6]

2 A farmer is investigating whether using a new fertiliser will increase the yield of tomato plants. The farmer selects 40 tomato plants at random and gives them the new fertiliser. The crop mass, x kg, of each of these 40 plants is recorded. The farmer selects a further 60 tomato plants at random and gives them a standard fertiliser. The crop mass, y kg, of each of these 60 plants is recorded. The results are summarised as follows.

$$\Sigma x = 168 \quad \Sigma x^2 = 720 \quad \Sigma y = 228 \quad \Sigma y^2 = 900$$

Find a 90% confidence interval for the difference in mean crop mass associated with each type of fertiliser. [7]

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3 A continuous random variable X has probability density function f given by

$$\mathrm{f}(x) = \begin{cases} kx & 0 \leqslant x < 1, \\ k(8-x) & 1 \leqslant x \leqslant 8, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(a) Show that $k = \frac{1}{25}$. [2]

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(b) Find the median value of X. [3]

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The random variable Y is defined by $Y = \sqrt[3]{X}$.

(c) Find the probability density function of Y. [5]

4 A researcher claims that older people take longer to react to a sudden loud noise than younger people. To investigate this, the researcher randomly selects 6 people over 50 years old and 8 people under 25 years old and records their reaction times, in milliseconds, to a sudden loud noise. The reaction times are as follows.

Over 50	198	212	217	229	235	242		
Under 25	178	181	183	192	203	209	223	231

Carry out a Wilcoxon rank-sum test at the 5% significance level to test the researcher's claim. [8]

5 A doctor is investigating the concentration of blood glucose in patients at risk of developing type 2 diabetes, where blood glucose is measured in appropriate units. The doctor claims that a particular intervention reduces the concentration by more than k units on average. A group of 8 at risk patients is selected at random and each patient follows the intervention for six months. The blood glucose concentrations before and after the intervention are given in the following table.

Patient	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*
Before	183	165	172	165	143	176	161	153
After	164	148	164	149	134	153	155	148

(a) Use a t-test at the 5% significance level to find the range of values of k for which the result of the test is to reject the null hypothesis. [7]

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(b) State an assumption necessary for the test in part **(a)** to be valid. [1]

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6 A bag contains 7 red balls and 3 blue balls. Kieran selects 2 balls at random, without replacement. The number of red balls selected by Kieran is denoted by X, and the number of different colours present in Kieran's selection is denoted by Y.

(a) Find the probability generating functions, $G_X(t)$ of X and $G_Y(t)$ of Y. [4]

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The random variable Z is the sum of the number of red balls and the number of different colours present in Kieran's selection. Kieran claims that the probability generating function of Z is equal to $G_X(t) \times G_Y(t)$.

(b) Explain why Kieran is incorrect. [1]

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(c) Find the probability generating function of Z, expressing your answer as a polynomial in t. [4]

(d) Use the probability generating function of Z to find $\mathrm{E}(Z)$. [2]

Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

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