Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

FURTHER MATHEMATICS **9231/44**

Paper 4 Further Probability & Statistics **May/June 2025**

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **12** pages.

[Turn over

1 A random sample of 12 observations of a normal random variable is taken. The results give unbiased estimates for the population mean and variance as 10.24 and 0.52 respectively.

Test, at the 10% significance level, the null hypothesis that the population mean is 10.6 against the alternative hypothesis that the population mean is less than 10.6. [4]

2 The level of sound produced by a particular type of machine was measured for a random sample of 11 such machines. The results, in suitable units, are shown below.

Machine	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*	*K*
Sound level	7.66	8.48	8.21	7.98	8.01	7.77	8.25	8.11	8.03	8.16	7.92

(a) Use a Wilcoxon signed-rank test to test whether the average sound level produced by this type of machine is more than 8.00. Use a 5% significance level. [6]

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(b) Give a reason why a Wilcoxon signed-rank test may be more appropriate than a *t*-test in this case. [1]

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3 A shop selling electrical goods has a team of three salespeople: Avril, Ben and Charlie. The manager wishes to investigate whether the salespeople are equally successful at selling particular types of items. The following table gives a record of a random sample of 250 sales of laptops, cameras and televisions, with the number sold by each of the three salespeople.

	Type of item			
	Laptop	Camera	Television	Total
Avril	31	40	24	95
Ben	23	45	29	97
Charlie	21	25	12	58
Total	75	110	65	250

Test, at the 10% significance level, whether there is independence between the type of item sold and the salesperson. [7]

4 The continuous random variable X has probability density function f given by

$$\mathrm{f}(x) = \begin{cases} kx & 0 \leqslant x < 1, \\ kx^2 & 1 \leqslant x \leqslant 2, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Show that $k = \frac{6}{17}$. [2]

(b) Find the cumulative distribution function of X. [3]

(c) Find the median value of X. [2]

(d) Find $\mathrm{E}\left(\frac{1}{X}\right)$. [2]

5 Eric has three identical coins, each of which is biased so that the probability of obtaining a head when it is thrown is $\frac{1}{3}$. The random variable X is the number of heads obtained when Eric throws the three coins at the same time.

(a) Find the probability generating function $G_X(t)$ of X. [2]

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Eric also has two fair 6-sided dice with faces numbered 1 to 6. The random variable Y is the number of sixes obtained when Eric throws the two dice at the same time. It is given that the probability generating function of Y is $\frac{25}{36} + \frac{10}{36}t + \frac{1}{36}t^2$.

Eric throws the three coins and the two dice. The random variable Z is the sum of the number of heads obtained and the number of sixes obtained.

(b) Find the probability generating function $G_Z(t)$ of Z, expressing your answer as a polynomial in t. [3]

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(c) Use $G_Z(t)$ to find $E(Z)$ and $Var(Z)$. [5]

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6 Lina and Mona are two statisticians who also write songs. The 'time' of a song is the number of minutes for which it lasts. For a random sample of 10 of her songs, Lina calculates a 95% confidence interval for the population mean time, μ minutes. This confidence interval is $2.95 \leqslant \mu \leqslant 3.13$. The times, x minutes, of Lina's songs are normally distributed.

(a) Find the values of Σx and Σx^2 for the 10 songs in Lina's sample. [5]

Mona's songs have times, y minutes, that are normally distributed. The times for a random sample of 8 of Mona's songs are summarised as follows.

$$\Sigma y = 24.8 \qquad \Sigma y^2 = 76.98$$

Mona claims that the population mean time of her songs is greater than the population mean time of Lina's songs.

(b) Assuming that the two distributions have the same population variance, test at the 5% significance level whether there is evidence to support Mona's claim. [8]

Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

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