Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

* 0 9 5 9 1 6 7 0 2 1 *

FURTHER MATHEMATICS **9231/12**

Paper 1 Further Pure Mathematics 1 **October/November 2025**

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

1 **(a)** Use standard results from the list of formulae (MF19) to find $\sum_{r=1}^{n}\left(r^3 - r\right)$ in terms of n, fully factorising your answer. [3]

(b) Express $\frac{r+3}{r^3-r}$ in the form $\frac{A}{r-1}+\frac{B}{r}+\frac{C}{r+1}$, where A, B and C are constants to be determined, and hence use the method of differences to find $\sum_{r=2}^{n}\frac{r+3}{r^3-r}$. [5]

(c) Deduce the value of $\sum_{r=2}^{\infty}\frac{r+3}{r^3-r}$. [1]

2 The cubic equation $x^3 + bx^2 + cx + d = 0$, where b, c and d are constants, has roots α, β and γ. It is given that

$$\alpha + \beta + \gamma = 2,$$
$$\alpha^2 + \beta^2 + \gamma^2 = 3,$$
$$\alpha^4 + \beta^4 + \gamma^4 = 5.$$

(a) Find the values of b and c. [3]

(b) Find the value of *d*. [5]

3 The sequence of positive numbers $u_1, u_2, u_3, \ldots$ is such that $u_1 < 5$ and, for $n \geqslant 1$,

$$u_{n+1} = \frac{6u_n + 5}{u_n + 2}.$$

(a) By considering $5 - u_{n+1}$, prove by mathematical induction that $u_n < 5$ for all positive integers n. [5]

(b) Show that $u_{n+1} > u_n$ for $n \geqslant 1$. [3]

4 Let k and m be non-zero constants. The matrices $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ are given by

$$\mathbf{A} = \begin{pmatrix} 0 & 1 \\ -1 & 1 \\ 1 & 1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} k & 0 \\ 0 & m \end{pmatrix} \text{ and } \mathbf{C} = \begin{pmatrix} 2 & -1 & 1 \\ 1 & 1 & 2 \end{pmatrix}.$$

(a) Give full details of the geometrical transformation in the x-y plane represented by the matrix $\mathbf{B}$ in each of the following cases.

(i) $m = 1$ [2]

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(ii) $m = k$ [2]

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(b) Show that the matrix $\mathbf{ABC}$ is singular. [6]

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5 The curve C has polar equation $r^2 = \tan 2\theta$, where $0 \leqslant \theta \leqslant \frac{1}{8}\pi$.

(a) Sketch C and state the greatest distance of a point on C from the pole. [2]

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(b) Find the exact value of the area of the region bounded by C and the half-line $\theta = \frac{1}{8}\pi$. [4]

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(c) Show that C has Cartesian equation $x^4 - 2xy - y^4 = 0$ given that $0 \leqslant x \leqslant \cos\left(\frac{1}{8}\pi\right)$ and $0 \leqslant y \leqslant \sin\left(\frac{1}{8}\pi\right)$. [4]

(d) Using your answer to **(b)**, deduce the exact value of the area bounded by C, the x-axis and the line $x = \cos\left(\frac{1}{8}\pi\right)$. [2]

6 The plane Π has equation $x + 3y + 2z = 1$.

(a) Find the perpendicular distance from the origin O to the plane Π. [2]

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Relative to O, the points A, B, C have position vectors

$$-\mathbf{j} + 2\mathbf{k}, \quad 2\mathbf{i} - \mathbf{k}, \quad 2\mathbf{i} - \mathbf{j} - \mathbf{k},$$

respectively.

(b) Find the acute angle between the planes OAB and Π. [4]

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(c) Find an equation for the common perpendicular to the lines OC and AB. [8]

7 The curve C has equation $y = \dfrac{x^2 + x + 1}{x + 1}$.

(a) Find the equations of the asymptotes of C. [3]

(b) Find the coordinates of any stationary points on C. [3]

(c) Sketch C. [3]

(d) Sketch the curve with equation $y = \dfrac{|x|^2 + |x| + 1}{|x| + 1}$. [2]

(e) Find, in exact form, the set of values of x for which $\dfrac{|x|^2+|x|+1}{|x|+1} < 3$. [3]

Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

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