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Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

* 3 1 9 0 1 5 7 7 1 4 *

FURTHER MATHEMATICS **9231/23**

Paper 2 Further Pure Mathematics 2 **October/November 2025**

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

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1 Find the values of k for which the system of equations

$$\begin{aligned} x - y + z &= 0, \\ x + ky + 3z &= 0, \\ x + 2y + kz &= 0, \end{aligned}$$

does not have a unique solution. [3]

2 The curve C has parametric equations

$$x = \sinh t \quad \text{and} \quad y = t + \cosh t.$$

(a) Find $\frac{dy}{dx}$ in terms of t. [2]

(b) Show that $\frac{d^2y}{dx^2} = \frac{1 - \sinh t}{\cosh^3 t}$. [4]

(c) Find the Maclaurin's series for y in terms of x up to and including the term in x^2. [2]

3 The integral I_n is defined by $I_n = \int_0^1 \left(1+x^2\right)^n \mathrm{d}x$.

(a) By considering $\frac{\mathrm{d}}{\mathrm{d}x}\left(x(1+x^2)^n\right)$, or otherwise, show that

$$(2n+1)I_n = 2^n + 2nI_{n-1}.$$ [5]

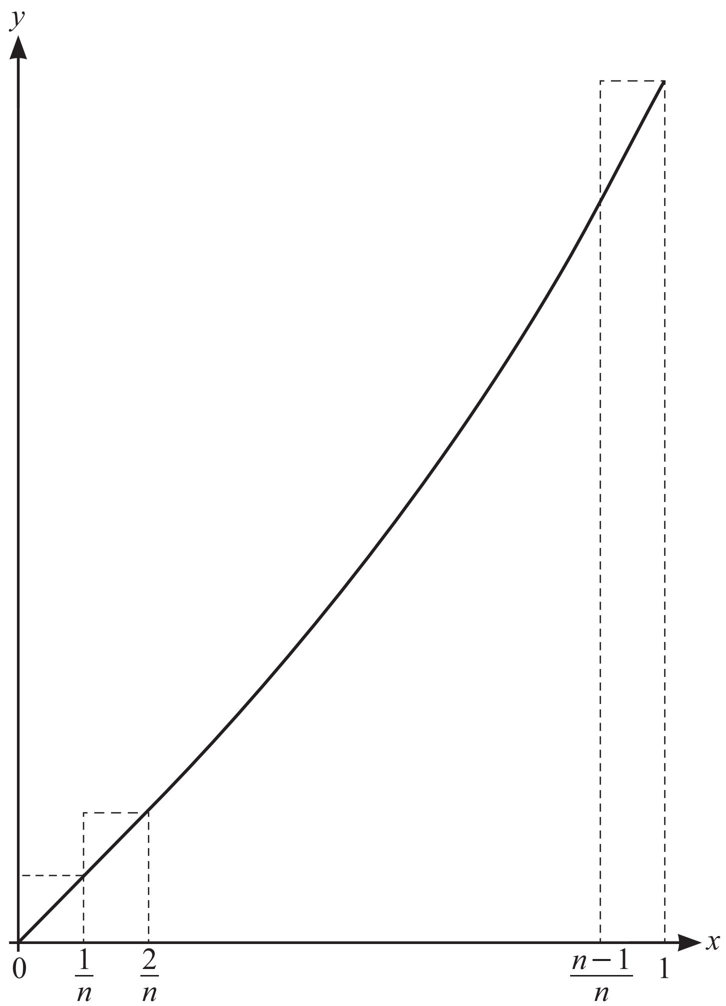
(b) Find the exact value of I_{-2}. [4]

4 Find the particular solution of the differential equation

$$5\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = x^2 + 5x + 3,$$

given that, when $x = 0$, $y = \frac{dy}{dx} = 0$. [10]

5



The diagram shows the curve with equation $y = \frac{1}{3}x^3 + x$ for $0 \leqslant x \leqslant 1$, together with a set of n rectangles of width $\frac{1}{n}$.

(a) By considering the sum of the areas of these rectangles, show that $\int_0^1 \left(\frac{1}{3}x^3 + x\right) dx < U_n$, where

$$U_n = \frac{1}{12}\left(1 + \frac{1}{n}\right)\left(7 + \frac{1}{n}\right).$$ [5]

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(b) Use a similar method to find, in terms of n, a lower bound L_n for $\int_0^1 \left(\frac{1}{3}x^3 + x\right) dx$. [4]

(c) Show that $\lim_{n \to \infty}(U_n - L_n) = 0$. [2]

6 The matrix **P** has non-zero eigenvalues and is given by

$$\mathbf{P} = \begin{pmatrix} a & 1 & 1 \\ 0 & 2a & -1 \\ 0 & 0 & -3a \end{pmatrix}.$$

(a) State, in terms of a, the eigenvalues of **P**. [1]

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(b) **(i)** Find $\mathbf{P}^2$ in terms of a. [1]

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(ii) Use the characteristic equation of **P** to find $\mathbf{P}^{-1}$ in terms of a. [3]

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The 3×3 matrix **A** has eigenvalues 1, 2, 3 with corresponding eigenvectors

$$\begin{pmatrix} a \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 2a \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ -3a \end{pmatrix},$$

respectively.

(c) Find **A** in terms of a. [5]

7 **(a)** Show that $\frac{\mathrm{d}}{\mathrm{d}x}\left(\tanh^{-1}x\right) = \frac{1}{1-x^2}$. [3]

(b) Find the solution of the differential equation

$$x\frac{\mathrm{d}y}{\mathrm{d}x} - y = x^2\tanh^{-1}x,$$

for $0 < x < 1$, given that $y = 0$ when $x = \frac{1}{2}$. Give your answer in an exact form. [9]

8 **(a)** State the sum of the series $z + z^2 + ... + z^n$, for $z \neq 1$. [1]

(b) By letting $z = \frac{1}{2}(\cos\theta + \mathrm{i}\sin\theta)$ use de Moivre's theorem to deduce that

$$\sum_{m=1}^{n}\left(\tfrac{1}{2}\right)^m \sin m\theta = \frac{\left(\tfrac{1}{2}\right)^{n+2}\sin n\theta - \left(\tfrac{1}{2}\right)^{n+1}\sin(n+1)\theta + \tfrac{1}{2}\sin\theta}{\tfrac{5}{4} - \cos\theta}.$$ [6]

(c) Use the result in **(b)** to find $\sum_{m=1}^{n}\left(\frac{1}{2}\right)^m m\cos m\theta$ in terms of n and θ. [You do not need to simplify your answer.] [3]

(d) Hence find $\sum_{m=1}^{\infty}\left(\frac{1}{2}\right)^m m\cos m\theta$ in terms of $\cos\theta$. [You may assume that $\frac{n}{2^n} \to 0$ as $n \to \infty$.] [2]

Additional page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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