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Cambridge Assessment International Education

# Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

*2123312647*

**FURTHER MATHEMATICS** **9231/24**

Paper 2 Further Pure Mathematics 2 **October/November 2025**

**2 hours**

You must answer on the question paper.

You will need: List of formulae (MF19)

**INSTRUCTIONS**

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

**INFORMATION**

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages.


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**[Turn over**

**1** It is given that $I_n = \int_0^1 x^n \sinh x \, \mathrm{d}x$, where $n$ is a non-negative integer.

**(a)** Show that, for $n \geqslant 2$,

$$I_n = \cosh 1 - n \sinh 1 + n(n-1)I_{n-2}. \quad [3]$$

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**(b)** Find the exact value of $I_2$. [2]

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**2** **(a)** Find the exact values of $x$ for which $\cosh 2x = 6\sinh^2 x$ , giving your answers in logarithmic form. [4]

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**(b)** Sketch the curves $C_1 : y = \cosh 2x$ and $C_2 : y = 6\sinh^2 x$ on the same diagram. [2]

**3** **(a)** Use de Moivre's theorem to show that

$$\sin 5\theta = 16\sin^5\theta - 20\sin^3\theta + 5\sin\theta.$$ [4]

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**(b)** Hence obtain the roots of the equation

$$32x^5 - 40x^3 + 10x - \sqrt{3} = 0$$

in the form $\sin q\pi$, where $q$ is a rational number. [4]

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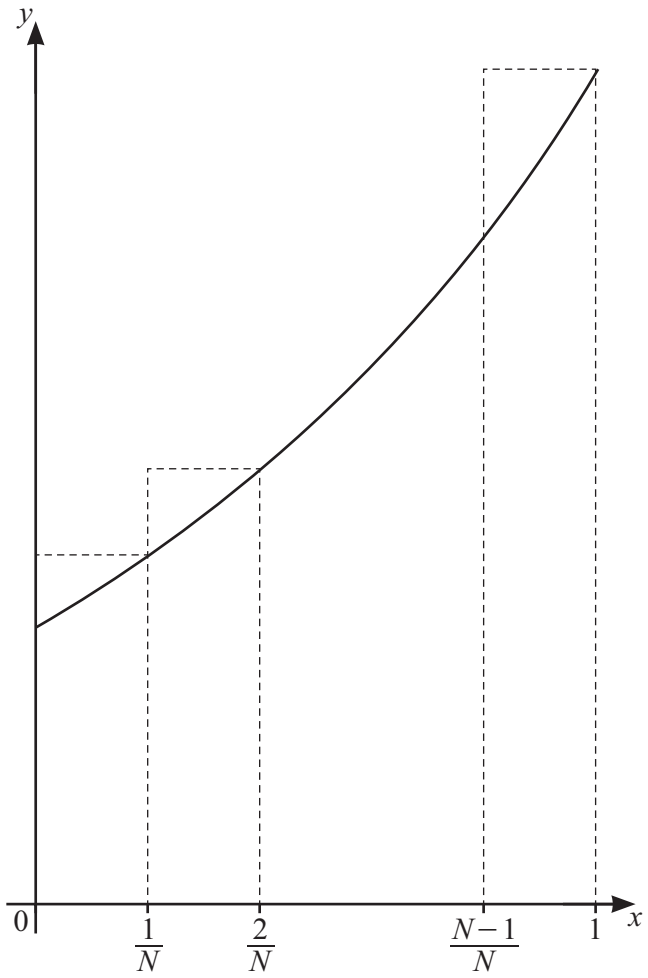


The diagram shows the curve with equation $y = \frac{1}{2}\left(3^x\right)$ for $0 \leqslant x \leqslant 1$, together with a set of $N$ rectangles each of width $\frac{1}{N}$.

**(a)** By considering the sum of the areas of these rectangles, show that $\int_0^1 \frac{1}{2}\left(3^x\right)\mathrm{d}x < U_N$, where

$$U_N = \frac{3^{\frac{1}{N}}}{N\left(3^{\frac{1}{N}} - 1\right)}.$$ [4]

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**(b)** Use a similar method to find, in terms of $N$, a lower bound $L_{\mathrm{N}}$ for $\int_0^1 \frac{1}{2}\left(3^x\right)\mathrm{d}x$. [4]

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**(c)** By simplifying $U_N - L_N$, show that $\lim_{N\to\infty}\left(U_N - L_N\right) = 0$. [2]

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**5** **(a)** Find the general solution of the differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+\frac{\mathrm{d}y}{\mathrm{d}x}+y=\sin 2x+\cos 2x. \qquad [7]$$

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**(b)** For large positive values of $x$ and for any initial conditions, show that the solution to part **(a)** can be approximated by

$$y \approx R\sin(2x-\phi),$$

where $R$ and $\phi$ are positive constants to be determined. [3]

**6** Find the particular solution of the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} + \frac{1+x}{1-2x-x^2}y = 1$$

given that $y = \pi$ when $x = 0$. Give your answer in an exact form. [10]

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**7** The curve $C$ has parametric equations

$$x = 8\ln(\tan\tfrac{1}{2}t) - 3\cot t - 3t, \quad y = 6\ln(\tan\tfrac{1}{2}t) + 4\cot t + 4t, \quad \text{for } \tfrac{1}{6}\pi \leqslant t \leqslant \tfrac{1}{3}\pi.$$

**(a) (i)** Show that $\dfrac{\mathrm{d}x}{\mathrm{d}t} = 8\,\mathrm{cosec}\,t + 3\cot^2 t$. [1]

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**(ii)** Find $\dfrac{\mathrm{d}y}{\mathrm{d}t}$. [1]

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**(iii)** Find the exact value of the length of $C$. [6]

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**(b)** Show that

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \frac{a\cot t\,\mathrm{cosec}\,t(\mathrm{cosec}^2t+1)}{(b\,\mathrm{cosec}\,t + c\cot^2 t)^n},$$

where $a$, $b$, $c$ and $n$ are integers to be determined. [5]

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**8** **(a)** Find the set of values of $a$ for which the system of equations

$$\begin{aligned} 5x + ay &= 3, \\ 20x - 5y &= 2, \\ 2x - 3y + z &= 1, \end{aligned}$$

has a unique solution and interpret this situation geometrically. [3]

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**(b)** Given that $a = -\frac{5}{4}$, show that the system of equations in part **(a)** is inconsistent and interpret this situation geometrically. [3]

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The matrix **A** is given by

$$\mathbf{A} = \begin{pmatrix} 5 & 0 & 0 \\ 20 & -5 & 0 \\ 2 & -3 & 1 \end{pmatrix}.$$

**(c)** Find a matrix **P** and a diagonal matrix **D** such that $(\mathbf{A} + 3\mathbf{I})^2 = \mathbf{PDP}^{-1}$. [7]

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## Additional page

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