Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

FURTHER MATHEMATICS **9231/33**

Paper 3 Further Mechanics **October/November 2025**

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use $10\,\text{m}\,\text{s}^{-2}$.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

1 Two uniform smooth spheres A and B of equal radii have masses $4m$ and m respectively. Sphere B is at rest on a smooth horizontal surface. Sphere A is moving on the surface with speed u and collides directly with sphere B. After the collision, the momentum of A is three times the momentum of B.

Find the value of the coefficient of restitution e. [4]

2 A particle P of mass m is moving in a horizontal circle with angular speed ω_1 on the smooth inner surface of a hemispherical shell of radius r. The angle between the upward vertical and the normal reaction of the surface on P is θ_1, where $\tan\theta_1 = \frac{3}{4}$.

When the angular speed is increased to ω_2, the angle between the upward vertical and the normal reaction of the surface on P becomes θ_2, where $\tan\theta_2 = \frac{4}{3}$.

Find the ratio $\dfrac{\omega_1}{\omega_2}$. [4]

3 A particle P is moving in a straight horizontal line. At time t s, the displacement of P from a fixed point O on the line is x m and the velocity of P is $v\,\mathrm{m\,s^{-1}}$. The acceleration of P is $\frac{1}{2}(v^2+4)\,\mathrm{m\,s^{-2}}$ in the direction PO. Initially P is at O and is moving with velocity $2\,\mathrm{m\,s^{-1}}$.

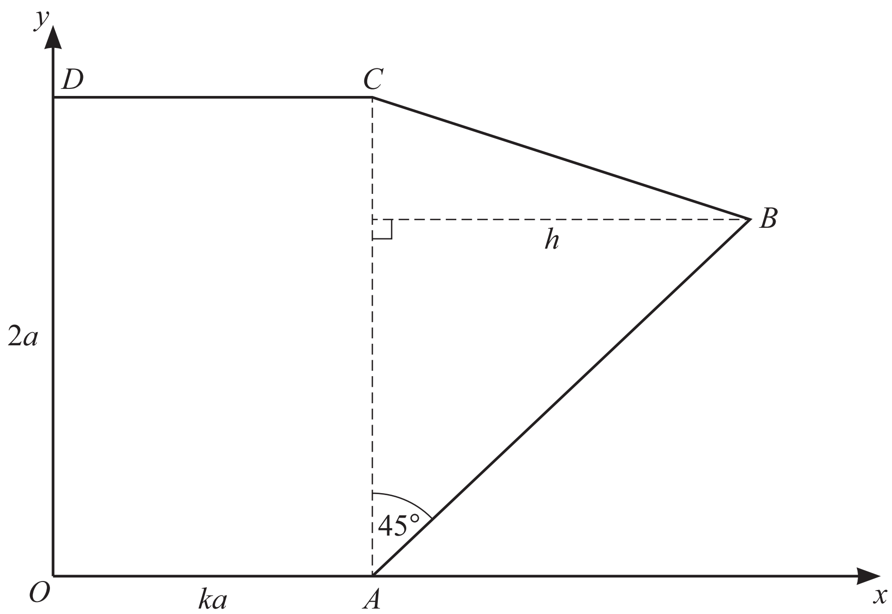
(a) Find an expression for x in terms of t. [5]

(b) Find the time when P next goes through O. [2]

4 One end of a light elastic string of natural length a and modulus of elasticity $5mg$ is attached to a fixed point O. Two particles, P and Q, of masses m and $4m$ respectively are attached to the other end of the string and they hang vertically in equilibrium. Particle Q is then detached from the string, hence releasing particle P from rest.

Find, in terms of a, the length of the string when the speed of particle P is first equal to $\sqrt{\frac{7}{5}ag}$. [6]

5



A uniform lamina $OABCD$ consists of a rectangle $OACD$ and a triangle ABC. The length of OA is ka, the length of OD is $2a$, the height of triangle ABC is h and angle CAB is 45° (see diagram). Relative to axes through O, parallel and perpendicular to OA as shown, the centre of mass of triangle ABC is $(\overline{x}, \overline{y})$.

(a) Show that $\overline{x}$ is $\frac{1}{3}(3ka+h)$, and find an expression for $\overline{y}$. [3]

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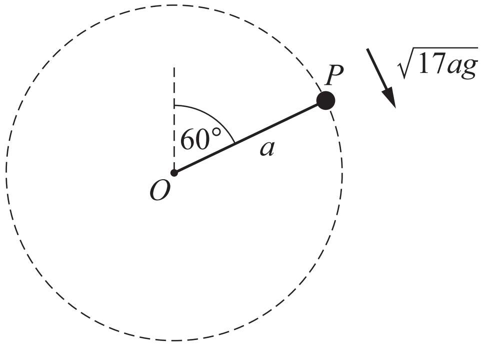
The lamina $OABCD$ is placed vertically on its edge OA on a horizontal plane.

(b) Find, in terms of a and k, the set of values of h for which the lamina is in equilibrium. [4]

It is now given that $k = \frac{\sqrt{3}}{3}$ and that the lamina is on the point of toppling.

(c) Find, in terms of a, the coordinates of the centre of mass of the triangle ABC. [2]

6



A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. Initially P is held with the string taut and making an angle of 60° with the upward vertical through O. The particle P is projected perpendicular to the string in a downwards direction with speed $\sqrt{17ag}$. It then starts to move along a circular path in a vertical plane with centre O (see diagram). At the lowest point of its path, vertically below O, the particle P collides with a stationary particle Q.

(a) Find, in terms of a and g, an expression for the speed of P immediately before the collision with Q. [2]

As a result of the collision, P rebounds and moves back along a circular path with centre O. The string becomes slack when P reaches the point on the circle vertically above O.

(b) Find, in terms of a and g, an expression for the speed of P immediately after the collision with Q. [3]

The mass of particle Q is km and the collision between P and Q is perfectly elastic.

(c) Find the value of k. [3]

7 A particle P is projected from a point O on a horizontal plane and moves freely under gravity. The initial velocity of P is $25\,\mathrm{m\,s^{-1}}$ at an angle θ above the horizontal, where $\tan\theta = \frac{4}{3}$. At point A, the direction of motion of P makes an angle of $45°$ with the downward vertical through A.

(a) By differentiating the equation of the trajectory or otherwise, find the coordinates of A. [5]

At point A, the particle strikes a fixed smooth barrier, rebounds, and lands on the horizontal plane. The barrier is inclined at an angle of 45° to the horizontal.

(b) Find the speed of P immediately before it collides with the barrier. [3]

(c) Given that the coefficient of restitution between the barrier and the particle is $\frac{1}{9}$, find the horizontal distance travelled by P after it strikes the barrier. [4]

Additional page

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